UNIVERSITI PUTRA MALAYSIA

QUANTUM COMPUTER ERROR CORRECTION

MUHAMMAD IZZAT BIN MUKTAR

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By

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DEDICATION

I dedicate my dissertation work to my family, my supervisor and many friends especially my group lab member. A special feeling of gratitude to my loving parents, Muktar Bin Sawi whose always pray for my success. Your words of encouragement and push for tenacity made me feel stronger day by day to complete my degree.

I also dedicate this dissertation to my supervisor, Dr Hishamuddin Zainuddin who have supported me throughout the process. I will always appreciate all she has done, especially her time for helping me finishing my final year project and her knowledge that she shared with me.

I dedicate this work and give special thanks to my friends for being there for me throughout the semester

ABSTRACT

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Ciri teori yang paling penting dalam pemprosesan maklumat kuantum sudah pasti pembetulan ralat kuantum. Dari permulaan bidang yang menarik ini, adalah jelas bahawa kerapuhan sistem kuantum yang koheren akan menjadi penghalang maut kepada pembangunan komputer kuantum berskala besar. Pembangunan Pembetulan Ralat Kuantum menunjukkan bahawa langkah aktif boleh digunakan untuk mengurangkan isu maut ini. Pembetulan Ralat Kuantum, sebaliknya, kini merupakan bidang yang lebih mantap, dengan banyak kod, kaedah dan pendekatan baharu dicipta untuk melakukan pembetulan ralat untuk algoritma kuantum skala besar. Kami berusaha untuk meringkaskan asas-asas Pembetulan Ralat Kuantum. Daripada memperkenalkan konsep ini melalui rangka kerja matematik dan sains komputer yang ketat, kami memeriksa pembetulan ralat terutamanya melalui contoh terperinci, berkembang daripada contoh asas kod 3-qubit kepada formalisme penstabil, yang kini amat penting apabila memahami struktur kod berskala besar yang digabungkan. dan litar kuantum.

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This study project has given me the opportunity to learn and gather knowledge that has helped me grow as a person both personally and professionally. I'd want to take this chance to express my gratitude to everyone who helped me till I was able to successfully complete this thesis.

First and foremost, I want to express my sincere gratitude to my supervisor, Prof. Madya Dr Hishamuddin Zainuddin, who has patiently provided me with direction and motivation throughout this project. His devotion, wise suggestions, and firm belief in my abilities assisted me greatly in finishing our final project.

Aside from that, I am grateful to my friends and course mates who were always there for me on my road to complete this task. I'd want to thank everyone who has generously shared any information or knowledge with me.

Last but not least, I'd like to show my gratitude to my family for their unwavering support and love for me. Without their unwavering support, this project would not be achievable.

Date: …………………………...……

Assoc. Prof.Dr Suriati Paiman

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CHAPTER 1

INTRODUCTION

1.1 Background of Study

Over the few years we have seen computers evolve drastically throughout the century. Now days our computer is far faster and better than the old computer that was first invented. All of this is possible due to the size of a transistor is getting smaller and smaller. This is very important in producing a better computer as smaller transistor means we can fit more transistors in a computer chip. However, as scientists are trying to reduce the size of a transistor it will finally come to the end of the road as transistors are reaching their physical limit in which any smaller size cannot be used as a switch anymore. Anything smaller than that will cause the transistor to obey a different set of physical laws called quantum mechanics where everything is opposite of the classical world. When the transistor is reduced to the near size of an atom, electrons may just transfer themselves to the other side of a blocked passage via a process called quantum tunnelling.

1.2 Problem Statement

There has been a lot of debating regarding the future of quantum computer. This is because quantum computer is very different to classical computer . The power of a quantum computer develops exponentially in proportion to the number of qubits coupled together. This contrasts with a classical computer, whose power grows in direct proportion to the number of transistors. From a quantum computing hardware perspective, the main issue is with the qubits themselves. By their very nature, they are error-prone and difficult to control, making quantum computers unstable and highly complex. Fortunately, something called quantum error correction exists, which is a type of algorithm that corrects the errors.

1.3 Objective of the Thesis

I. To understand the basic structure of quantum mechanics.
II. To identify the type of error in quantum computer.
III. To solve quantum error using different code.

1.4 Scope of the Thesis

This thesis consists of five chapters. The first chapter begins with an introduction to the scope of our main problems. The problem statement and research objective are also included.

Then chapter 2 give an overview of what is quantum mechanics and what are the properties of quantum mechanics. This chapter will also introduce the principle of quantum error correction as well the progression of quantum computer.

Chapter 3 will introduce to the basic structure of quantum computer error correction where this kind of code are the basic structure for a full quantum error correction code. In this chapter we will discuss how to fix error for a bitflip and phase plip error as well as introducing a stabilizer code which is the fundamental to nearly all quantum error correction code.

Next for chapter 4 we will introduce to the full quantum error correction, the 9-qubit code by Peter Shor and the surface code . this code can fix a continues error that will happen in quantum computer. Lastly in chapter 5 the application of quantum computer is introduced and suggestion for future research are also included.

CHAPTER 2

LITERATURE REVIEW

2.1 Quantum Bit (qubit)

If you ever heard or read anything to do with quantum computer, you are certain to stumble across the word 'qubit'. Every major advancement in the development of quantum computers appears to concentrate around adding more qubits, making them more stable and less 'noisy.' But what exactly does this mean?

First, we need to know what a qubit is. A qubit is the smallest unit in quantum mechanics. If in classical computer this information consists of 0 and 1 but in quantum, qubit consists of two state which are 0 and 1 . According to Brun, (n.d.) this qubit state can be represented as the form of matric as below :

$$0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad 1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Qubits are very different from classical bits as they can be in a state of both $|1\rangle$and $|0\rangle$ at the same time. It is said that qubit can be in a superposition of $|0\rangle$ and $|1\rangle$ at the exact same time until it is measured and are forced to give only one value. Superposition can be described as $\alpha\,|0\rangle + \beta\,|1\rangle$. According to Marquezino et al., (2004) normalization of this equation will result in $|\alpha|^2 + |\beta|^2 = 1$ in which both α and β are complex parameters. Below is the step to prove the normalization of the above equation.

2.2: The Bloch Sphere

As up to now we have represented qubit in a 2-dimensional space. But then question arise is to how we can apply this to the real world where everything is in a 3 dimensional? the answer to that is we can refer to the Bloch sphere model.

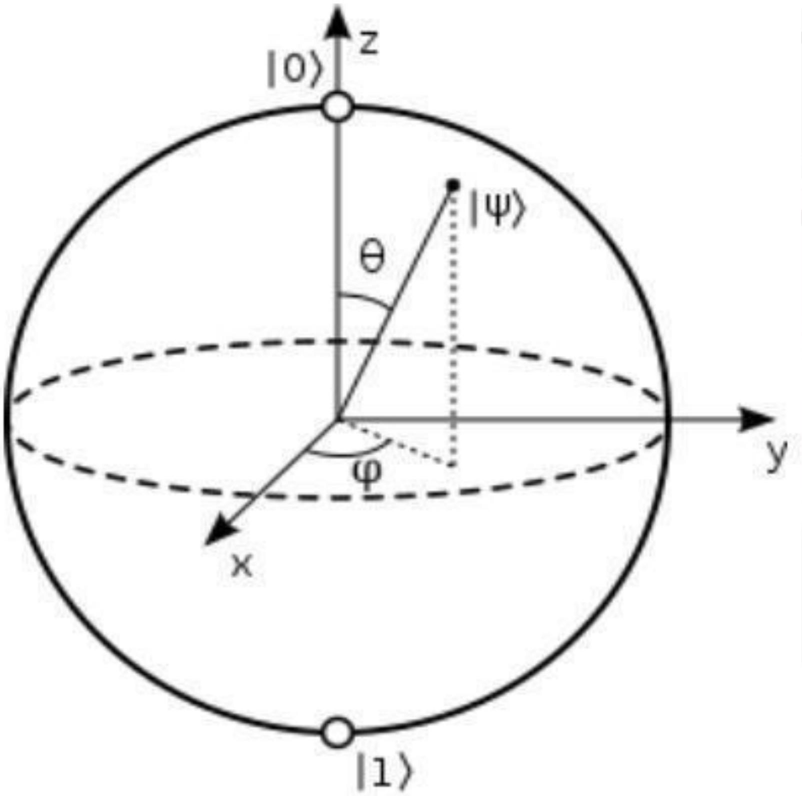


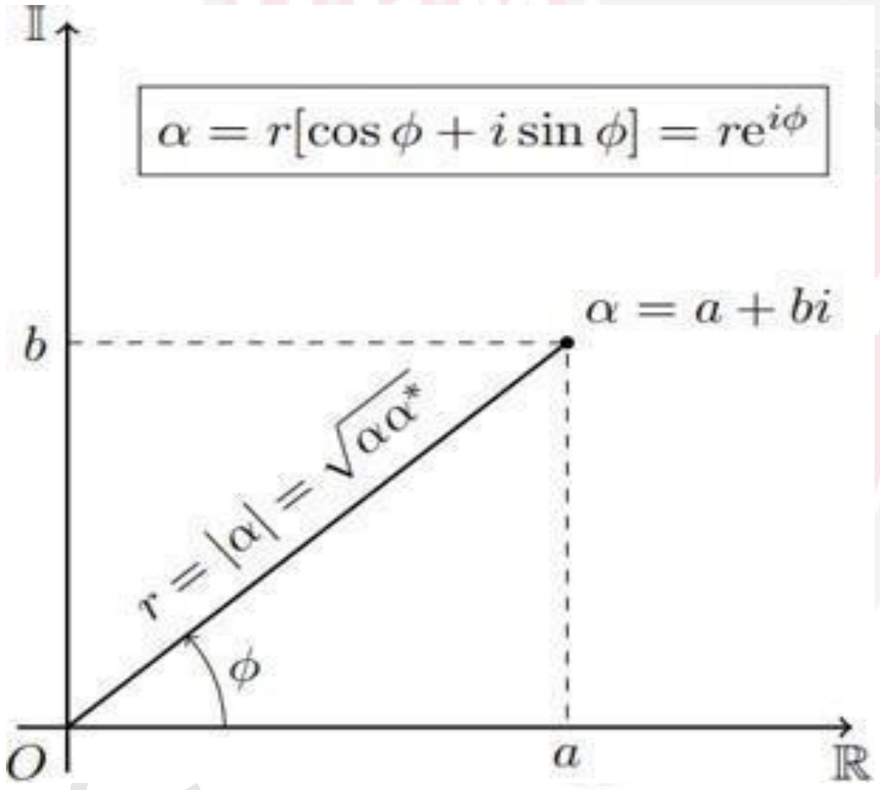
Figure 2.1 : The Bloch sphere (Meyer, n.d.-a)

Bloch sphere that was discovered by a great scientist name Felix Bloch are just another way of representing a normalized single qubit of two-level system. An example of a two-level system is a spin ½ particle of an electron. These two bases are commonly placed at the north and south poles of the Bloch sphere, and each other point on its surface corresponds to a specific

Superposition of $|0\rangle$ and $|1\rangle$ that can be fine at the surface of the Bloch sphere. It should be noted that this is only true for pure quantum states, whereas mixed are contained inside the Bloch sphere. For this Bloch spheremodel, we only need two parameters which is θ and φ. Below are example how a single qubit can be represented in Bloch sphere (refer equation 2.3):

$$|\Psi\rangle = \cos\frac{\theta}{2}\ 0\ + e^{(i\phi)}\sin\frac{\theta}{2}\ 1$$

Below is the derivation for this equation



$$\alpha = a + ib$$

$$= r\cos\phi + ir\sin\phi$$

$$\rightarrow a = r\cos\phi \text{ and } b = r\sin\phi$$

$$= r\left[\cos\phi + i\sin\phi\right]$$

$$\rightarrow: e^{i\phi} = r\cos\phi + i\sin\phi$$

$$= r\,e^{i\phi}$$

Now let $\alpha = r_0 e^{i\phi_0}$ and $\beta = r_1 e^{i\phi_1}$. Then

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = r_0 e^{i\phi_0}\,|0\rangle + r_1 e^{i\phi_1}\,|1\rangle$$

We can simplify the equation to

$$|\psi\rangle = e^{i\phi_0}[r_0\,|0\rangle + r_1 e^{i(\phi_1-\phi_0)}\,|1\rangle]$$

According to Meyer, (n.d.) $e^{i\phi_0}$ will not be needed so we can just remove this to

$$|\psi\rangle = e^{i\phi_0}[r_0\,|0\rangle + r_1 e^{i(\phi_1-\phi_0)}\,|1\rangle]$$

Now, one can define , $\phi = \phi_1 - \phi_0$ .Then what we got is

$$|\psi\rangle = r_0\,|0\rangle + r_1 e^{i\phi}\,|1\rangle$$

But remember we have a normalization condition in which

$$r_0^2 + r_1^2 = 1$$

Equivalently we can write

$$r_0 = \cos \theta/_2$$
$$r_1 = \sin \theta/_2$$

Now substitute this value and we finally get

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{(i\phi)}\sin\frac{\theta}{2}|1\rangle$$

At the x axis

$$\begin{aligned}
&x_+ : \theta = \frac{\pi}{2}, \phi = 0 \longrightarrow \\
&\cos\frac{\pi}{4}|0\rangle + e^{i\cdot 0}\sin\frac{\pi}{4}|1\rangle \\
&= \frac{1}{\sqrt{2}}|0\rangle + (1)\frac{1}{\sqrt{2}}|1\rangle \\
&= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\
&\equiv |+\rangle
\end{aligned}$$

$$\begin{aligned}
&x_- : \theta = \frac{\pi}{2}, \phi = \pi \longrightarrow \\
&\cos\frac{\pi}{4}|0\rangle + e^{i\pi}\sin\frac{\pi}{4}|1\rangle \\
&= \frac{1}{\sqrt{2}}|0\rangle + (-1)\frac{1}{\sqrt{2}}|1\rangle \\
&= \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \\
&\equiv |-\rangle
\end{aligned}$$

At the y axis

$$y_+ : \theta = \frac{\pi}{2}, \phi = \frac{\pi}{2} \longrightarrow$$
$$\cos\frac{\pi}{4}|0\rangle + e^{i\frac{\pi}{2}}\sin\frac{\pi}{4}|1\rangle$$
$$= \frac{1}{\sqrt{2}}|0\rangle + i\frac{1}{\sqrt{2}}|1\rangle$$
$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

$$y_- : \theta = \frac{\pi}{2}, \phi = \frac{3\pi}{2} \longrightarrow$$
$$\cos\frac{\pi}{4}|0\rangle + e^{i\frac{3\pi}{2}}\sin\frac{\pi}{4}|1\rangle$$
$$= \frac{1}{\sqrt{2}}|0\rangle - i\frac{1}{\sqrt{2}}|1\rangle$$
$$= \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

At the *z* axis

$$z_+ : \theta = 0, \phi = 0 \longrightarrow$$
$$\cos\frac{0}{2}|0\rangle + e^{i0}\sin\frac{0}{2}|1\rangle$$
$$= \cos 0\,|0\rangle + (1)\sin 0\,|1\rangle$$
$$= 1\,|0\rangle + 0\,|1\rangle$$
$$= |0\rangle$$

$$z_- : \theta = \pi, \phi = 0 \longrightarrow$$
$$\cos\frac{\pi}{2}|0\rangle + e^{i0}\sin\frac{\pi}{2}|1\rangle$$
$$= \cos 0\,|0\rangle + (1)\sin\frac{\pi}{2}|1\rangle$$
$$= 0\,|0\rangle + 1\,|1\rangle$$
$$= |1\rangle$$

2.3: Quantum Entanglement

Spin is a feature shared by all fundamental particles; its property is comparable to, but not identical to, that of the angular momentum in classical physics. In space, spin has a direction. Observers must choose a direction to measure the spin, and there are only two possible outcomes: spin up (in the same direction as the measurement) or spin down (opposite direction). When a pair of electrons are generated, interact, or share special proximity, their spin states can get entangled in which scientist call this phenomenon as quantum entanglement of electron.

Figure 2.3 : quantum entanglement

According to Xue (2021), the two particles are both in a superposition state, which means their spins are exactly opposite each other and their spin directions remain unknown until one of them is measured. It is not liked a pair of shoes where we know one is on the left and the other must be on the right since the characteristics of each shoe have been known earlier.

Changing the state of an entangled qubit instantaneously changes the state of the paired qubit in quantum computers. As a result, entanglement accelerates the processing speed of quantum computers. Because processing one qubit reveals information about numerous qubits, doubling the number of qubits does not necessarily increase the number of processes (i.e. the entangled qubits). Quantum entanglement, according to studies, is required for a quantum algorithm to deliver an exponential speedup over classical calculations.

2.4 No cloning theory

According to Fan et al., (2013) the ability to clone classical information is a critical feature. Cloning, or copying, appears to be unproblematic in classical systems. Information saved on computers can simply be duplicated for backup purposes. However, this is not the case with quantum systems. The no cloning theorem are very important in quantum mechanics as it forbids us from using conventional error correcting techniques to quantum systems. We cannot, for example, make backup copies of a state in the middle of a quantum computation and utilize them to correct if any errors have occurred. This theorem, on the other hand, is a critical component of quantum cryptography because it prevents eavesdroppers from generating duplicates of a transmitted quantum cryptographic key. The no-cloning theorem fundamentally safeguards the uncertainty principle in quantum physics. If one could clone an unknown state, one could produce as many copies as one wanted and measure each dynamical variable with arbitrary accuracy, thereby circumventing the uncertainty principle. The non-cloning theorem prevents this.

2.5 Quantum Error Correction Obstacle

At first a lot of scientists think that building a quantum computer is absurd, crazy and would never happen in a million years. This is because there are a lot of aspect that one need to really consider such as :

A. Error correction is critical. Error correction is required in quantum computing as the physical systems for a single qubit are extremely tiny which mean any slight outside interference can destroy the quantum state instantly .
B. Measurement of qubit destroy quantum information. Any attempt to measure the quantum information will destroy the superposition of the qubit which are what we are trying to preserve .
C. Errors of a broader kind can arise. Bit flips are not the only type of mistake that can occur. For example, phase error can also occur.

2.6: Principles of Error Correction

Now before we do any quantum computer error correction the first objective is to determine the qualities of the physical systems that can represent and compute with information. As a result, it is required to study the following:

A. Determine a code, which is a physical system subspace that may represent the information to be processed.
B. Determine a decoding technique capable of restoring the information contained in the code after any of the most likely error happened.
C. Alternatively, identify a pair of syndrome and information-carrying subsystems in which the code corresponds to a "base" state of the syndrome subsystem and the primary errors only affect the syndrome.
D. Examine the code's and subsystem's error behavior.

The duties of establishing a code and finding decoding techniques or subsystems are closely connected. As a result, the following questions form the basis of error-correction theory: What features must a code have in order to be effective in protecting against a certain error model? How does one receive the decoding or subsystem identification required for this protection? In many circumstances, the answers can be based on selecting a predefined set of error operators that accurately reflects the most likely error and then analyzing whether these errors can be safeguarded against without any loss of information. Once an error set has been corrected, assessing whether it is "correctable" may be expressed in terms of "detectable" faults (Knill et al., 2002).

2.7 Progression of quantum computer

However, as progression are moving, there seem to be a light at the end of the tunnel as scientist has finally discover a code that can fix quantum computer error. One of the examples is Peter Shor who come out with a 9-qubit code that can fix a single error and the surface code that can fix a continue errors. The success of this code has opened the eyes of many great scientists, mathematicians and also engineers to further study and explore this unusual quantum principle.

In 2019, International Business Machine (IBM) has released the 27-qubit falcon processor, which introduced the heavy hex qubit arrangement, which places qubits on the edges and corners of hexagons to decrease errors caused by inter-qubit interference.

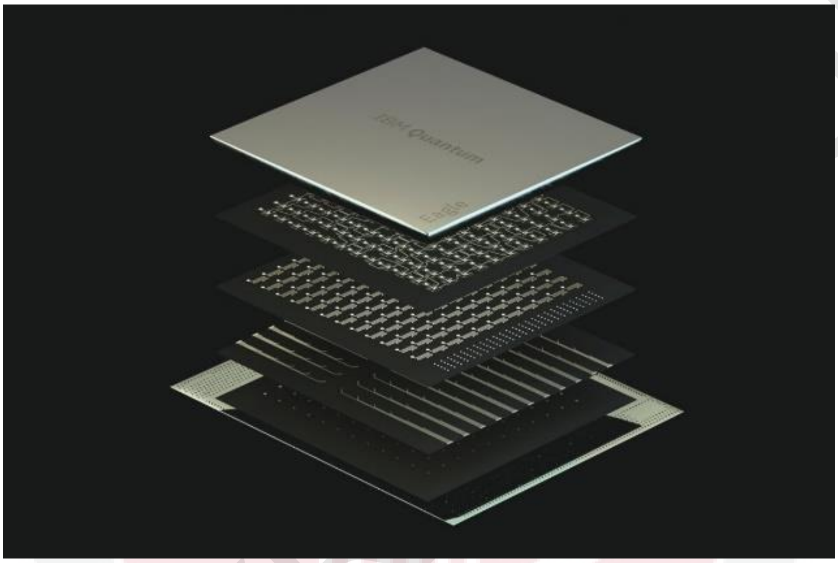


Figure 2.4 : Engle 127-qubit chip produce by IBM

Now IBM has announced the first commercial quantum processor to cross the 100-qubit barrier. Eagle is a new 127-qubit chip. Scalability is also dependent on quantum processor packing. However, a quantum chip usually needs a tangle of wires that must be directed outward to the chip's edge. But now with the advance technology and rapid research it allows for the incorporation of 3D integration into the Eagle. This will enable the chip components and wiring to be placed on many physical layers, making the road to a 1000 qubit quantum computer appear achievable. In the future, IBM intends to deliver the Osprey, a 433-qubit CPU, as well as the Condor, a 1,121-qubit CPU. This demonstrates that in the future, quantum computers will be the next supercomputer capable of solving problems quickly.

CHAPTER 3

THEORY AND METHODOLOGY

3.1 : Quantum gate

1) The first quantum gate is called NOT gate. Sometime this gate is also called Pauli-X gate. These NOT gate will change 1 to 0 and vice versa.

Gate	Matrix
	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Table 3.1 : The symbol for NOT gate and its matric form

Below show how the NOT gate work in matric form.

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

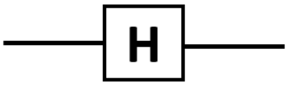
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Remember that :

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2) The second gate is called the Hadamard gate. The Hadamard gate allows us to go away from the Bloch sphere's poles and produce a superposition of $|0\rangle$ and $|1\rangle$.

Gate	Matrix
	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Hadamard gate maps the basis state :

$$|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \qquad |0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

One of the interesting properties of a Hadamard gate is if we apply Hadamard gate twice to aqubit the state would not change at all.

Below are the example ,

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Applying our first Hadamard gate

$$H|\Psi\rangle = \alpha\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + \beta\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$= \frac{1}{\sqrt{2}}\left[(\alpha+\beta)|0\rangle + (\alpha-\beta)|1\rangle\right]$$

Now we apply our second Hadamard gate :

$$\frac{1}{2}\left[(\alpha+\beta)(|0\rangle+|1\rangle)+(\alpha-\beta)(|0\rangle-|1\rangle)\right]$$

$$\frac{1}{2}\left[(\alpha+\beta+\alpha-\beta)|0\rangle+(\alpha+\beta-\alpha+\beta)|1\rangle\right]$$

3) The third gate is called CNOT gate. This gate is often called controlled Pauli-x gate due to this gate operation are nearly the same as the Pauli-X gate.

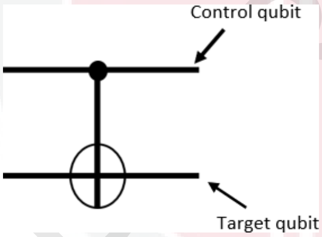
Gate	Matrix
	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Table 3.3 : The symbol for Hadamard gate and its matric form

The CNOT gate leaves the control qubit unchanged and performs a Pauli-X gate on the target qubit when the control qubit is in state |1⟩ and leaves the target qubit unchanged when the control qubit is in state |0⟩.

3.2 Classical Error Correction

The identification and correction of errors is an important topic in computer science. If we want to create a computer, whether it's a classical or quantum-mechanical one, we must first deal with its errors in order to trust these computations. We can only consider one type of error in a classical computer. Let’s say we want to send information through a channel. If this channel has noise, a bit 0 can be changed to 1 and a bit 1 to 0 with probability . For a classical error correction, we can use the repetition method in which we can copy our bit of information as shown below

$$0 \rightarrow 000$$

$$1 \rightarrow 111$$

A simple way of understanding this is imagine if we want to transfer a bit of information let’s say 1 through a channel to another person . Now if the channel has noise or if we misheard the information, we could easily receive the wrong input. However, if we triple this information by copying this 1 bit of information to 111, we can easily detect this error. Now let’s say if a error has occur and now this 111 bit has somehow change to 101. By using a majority vote, we can recover this information because there are more 1 than 0 which obviously, we can tell that the original input was 0 before error has occur. Majority vote in this example mean that the one with the most numbers among the bits of information will be the final output.

3.3 : The 3-qubit code

To start with our quantum computer error correction, we will firstly go to the basic of quantum error correction in which we will try to fix a bit flip error . Now if only one error has occurred, we can fix this error by applying our classical error correction technique. The 3-qubit bit flip algorithm is the most fundamental to understanding quantum error correction. It is to be highlighted that this code cannot fully fix quantum computer error due to this code cannot effectively correct for bit flip and phase flip at the same time. It is to be noted that when we refer to a 3 identical qubit, we are referring to a logical qubit in which :

$$|1\rangle_L = |111\rangle \qquad \text{and} \qquad |0\rangle_L = |000\rangle$$

Physical qubits are the number of qubits on the box in our computer, but logical qubits are groups of physical qubits that we employ as a single qubit in our calculation. The circuit shown in figure 8 has the ability to triple a single qubit. Based on figure 8 the input which is denoted by $|x\rangle$ can be anything but for now firstly we are going to see what will happen if we set the value of $|x\rangle = |0\rangle$. Then, because none of the two CNOT gates act on qubit $|0\rangle$, the final product will result in three qubits that is $|000\rangle$. Now if we change the value of $|x\rangle$ to $|1\rangle$ both of this CNOT gate will act on the target qubit $|0\rangle$ and flip this qubit to $|1\rangle$ which produce three qubits of $|111\rangle$. It is to be noted that a CNOT gate will only flip a qubit if the control qubit is $|1\rangle$.

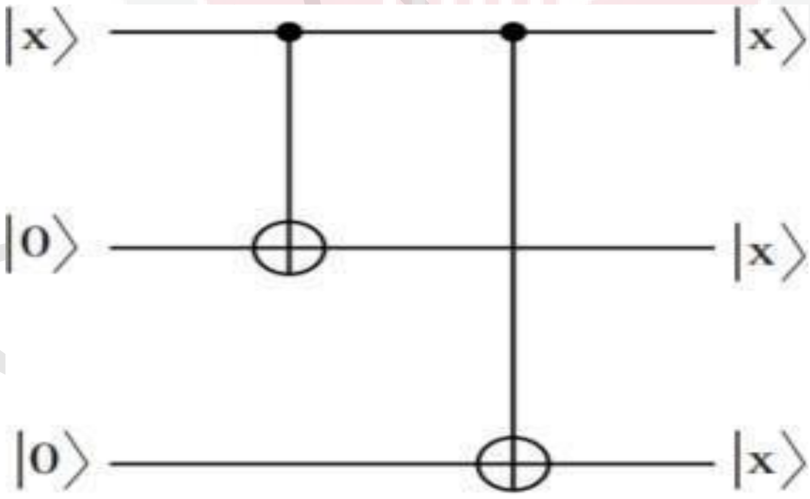


Figure 3.1 : Circuit to encode 3 qubits of its input (Young, n.d.)

The tensor product of the qubit in Figure 3.1 is as below :

$$|0\rangle \otimes |0\rangle = |0\rangle|0\rangle = |00\rangle$$

$$|00\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} \quad = \quad \begin{tikzcd} & \begin{pmatrix}1\\0\end{pmatrix} \\ \begin{pmatrix}1\\0\end{pmatrix} \arrow[ur] \arrow[dr] & \\ & \begin{pmatrix}1\\0\end{pmatrix} \end{tikzcd}$$

$$= \begin{pmatrix}1\times 1\\0\times 1\\0\times 1\\0\times 0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

This state of 1 and 0 can also be transformed into a linear combination that is :

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha\,|0\rangle_L + \beta\,|1\rangle_L = \alpha|000\rangle + \beta|111\rangle$$

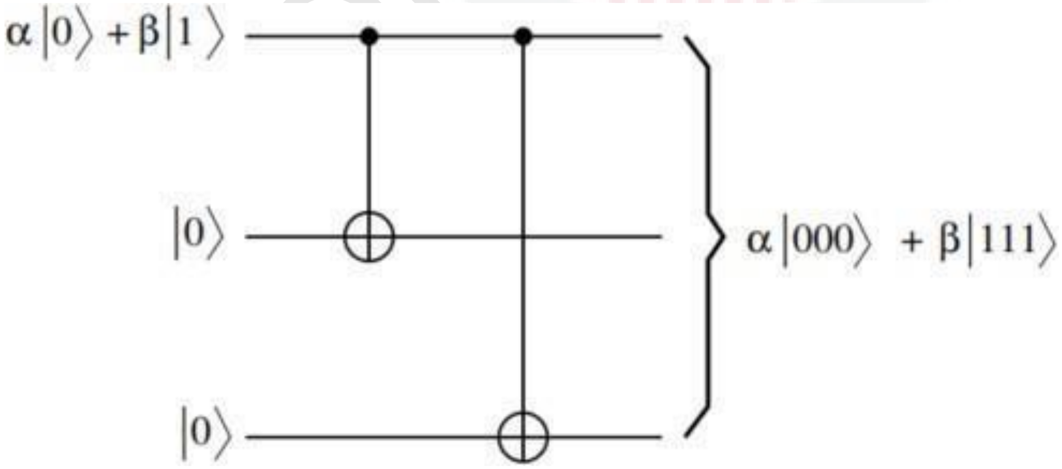


Figure 3.2 : Encoding circuit to produce 3 qubits based on a linear combination of 0 and 1 (Young, n.d.)

Table below show all the 8 possible states that error can occur with its probability. Based on the table p is the probability of one qubit being flipped and it is to be noted that flipping different qubit are independent with each other.

Table3.4 : The probability for each state of the syndrome

State	**probability**
$\alpha\ 000 + \beta\ 111$	$(1-p)^3$
$\alpha\ 100 + \beta\ 011$	$p(1-p)^2$
$\alpha\ 010 + \beta\ 101$	$p(1-p)^2$
$\alpha\ 001 + \beta\ 110$	$p(1-p)^2$
$\alpha\ 110 + \beta\ 001$	$p^2(1-p)^2$
$\alpha\ 101 + \beta\ 010$	$p^2(1-p)^2$
$\alpha\ 011 + \beta\ 100$	$p^2(1-p)^2$
$\alpha\ 111 + \beta\ 000$	p^3

When we have finish encode our input qubit, how do we know if our qubit in figure 3.2 has an error occurring in either of these three qubits? The answer for that question will be explain in the next section.

3.4 : The 3-qubit code error correction

Traditionally, we could just check at the bits to see if one of them was flipped. However, if we measure this qubit, we will break the coherent superposition. As a result, quantum error correction may appear to be impossible.

Surprisingly, this is not the case. The key is to connect our qubits with two extra qubits known as ancilla qubit and only measure these two qubits. Information from these two qubits can give us the information that we need to fix if any error has occurred without disrupting the coherent superposition.

The aim will be to discover whether any have flipped, which ones have, and then repair the issue. As a result, there will be either no error has occurred, or one error has occurred among these 3 qubits. These can be summarized as the following

$\alpha\|000\rangle + \beta\|111\rangle$	No error has occurred
$\alpha\|100\rangle + \beta\|011\rangle$	Error occurs at the first qubit
$\alpha\|010\rangle + \beta\|101\rangle$	Error occurs at the second qubit
$\alpha\|001\rangle + \beta\|110\rangle$	Error occurs at third qubit

These four states are referred to as "syndromes." It is worth mentioning that we refer to the left-hand qubit as the first qubit, the one to its right as the second qubit, and so on, for example $|x_1 x_2 x_3\rangle$

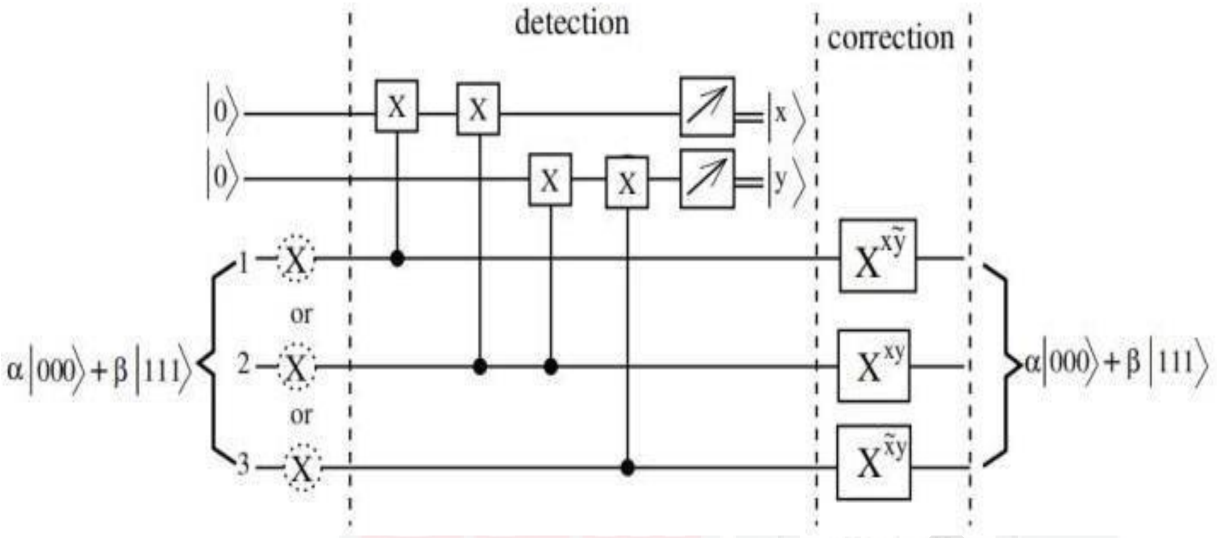


Figure 3.3 : Full circuit code for 3-qubbit code error correction (Young, n.d.)

For a 3-qubit error correction code we will need two extra qubits known as ancilla qubit. This qubit is at the top of the circuit while the three qubit that we want to know if error has occurred are at the bottom. It is to be noted that we are only measuring the ancilla qubit in which the first qubit will give the value of x and the second qubit will give the value for y. Now we will continue how we can get the four syndrome that we have stated.

Codeword qubits 1 and 2 are aimed towards the first (upper) ancilla (x).

Codeword qubits 2 and 3 target the 2nd (lower) ancilla (y)

1) For codeword 000 , both ancilla 0 do not flip. hence the value for x= 0 and y= 0

For codeword $|111\rangle$ the first and second ancilla will be flipped to $|1\rangle$ and again the ancilla qubit will be flipped to $|0\rangle$. Hence the value for $x = 0$ & $y = 0$.

Based on this result, we can see that if all the qubit is $|000\rangle$ or $|111\rangle$ the measurement of the ancilla will always be 0.

2) For codeword $|100\rangle$, the first ancilla qubit will be flipped to $|1\rangle$ but the second ancilla qubit will not be flipped . Hence the value of x= 1 & y=0.

   For codeword $|011\rangle$ the first ancilla qubit will be flip to $|1\rangle$ but the second ancilla qubit will be flip twice. Hence the value of x= 1 & y=0.

3) For codeword $|010\rangle$ both of the ancilla will be flipped once. Hence the value of x= 1 & y=1.

   For codeword $|101\rangle$ both of the ancilla will be flipped once. Hence the value of x= 1 & y=1.

4) For codeword $|001\rangle$ the first ancilla did not flip but the second ancilla is flipped once. Hence the value of x= 0 & y=1

   For codeword $|110\rangle$ the first ancilla qubit will be flipped to $|1\rangle$ and then it will be flipped again to $|0\rangle$ while the second ancilla will only be flipped one time . x= 0 & y=1

3.5 : Phase flip code

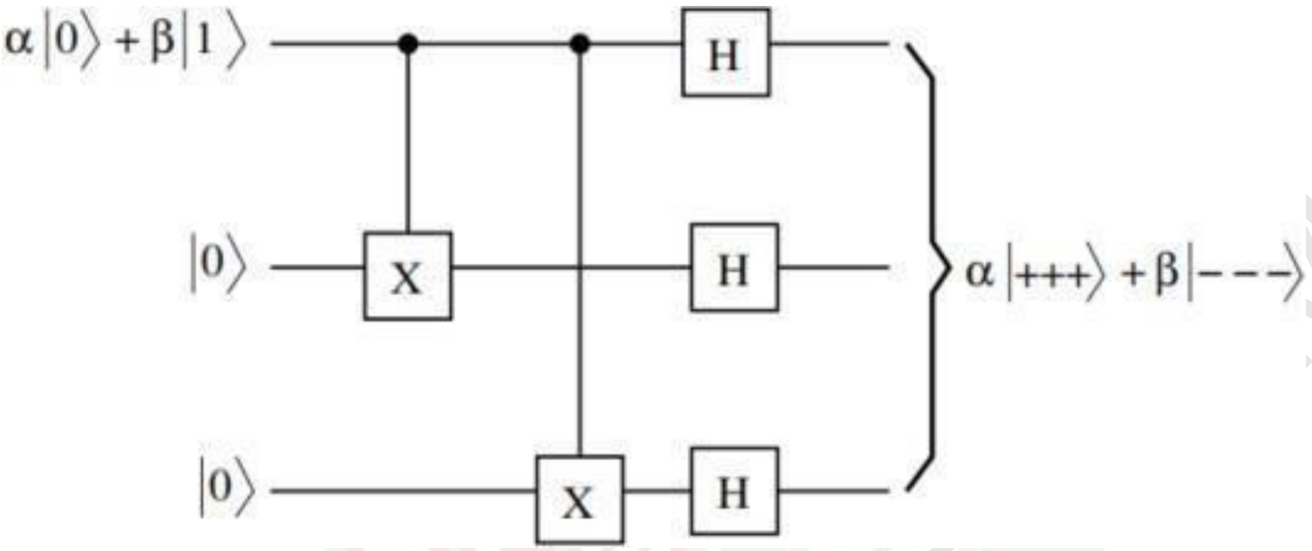


Figure 3.4 : Circuit to fix a phase flip error

Since we already review error correction for a bit flip error, we can apply this concept to a phase flip. We can change to the phase flip basis as

$$|+\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

It is to be noted that the function of the x and z gates have been swapped

$$X|0\rangle = X|1\rangle \qquad X|1\rangle = X|0\rangle$$

$$Z|0\rangle = X|0\rangle \qquad Z|1\rangle = Z-|1\rangle$$

3.6 : Stabilizer Code Up

Up to now, we have reviewed different type of code to fix quantum error computer in terms of their state representations, as well as their preparation and repair circuits. Most of the codes that is stated in this review are classified as stabilizer codes. Once the code's stabilizer structure is given, the general formalism is widely applicable, and there are universal rules for creating circuit and to correct the circuit. . Now at the beginning we express quantum state in form of basic, but a well-known scientist name Daniel Gottman came out with a clever idea of instead represent this state in basic, we can represent in the form of operators that is known as stabilizer formalism.

Look at the two Hermitian operators z_1z_2 and z_2z_3. Because z_i^2 =1and that different commute we then have

$$(Z_1Z_2)^2 = 1 \quad and \quad (Z_2Z_3)^2 = 1$$

Because acting twice on an eigenvector with the operator generates the eigenvector, the square of the eigenvalue is one, an operator whose square is unity has eigenvalues equal to one. We also know that z_1z_2 and $z_2 z_3$ commute together.

Any state that has no error has eigenvalue +1 for both stabilizers which is a fundamental characteristic that all stabilizers must possess. It should be noted that $z_1z_2 = 1$ corresponds to $x = 0$, whereas $z_2z_3 = 1$ corresponds to $x = 1$.

Chapter 4 :

Results and Discussion

4.1 : Quantum Error Detection

So far, we have emphasized on the capabilities to not only detect but also fix errors. Another option is to avoid applying the correction requirement. Knell's post-selected quantum computation revealed that large scale quantum computing may be performed with substantially greater noise rates when error detection is used instead of more expensive correction protocols (Knill, n.d.). Instead of doing active correction, the central concept of post-selected systems is to encode the computer with error detecting circuits, and if errors are found, the relevant subroutine of the quantum algorithm is reset and performed again. One disadvantage of these systems is that, while they result in low tolerable error rates, the resource needs are unrealistically high.

The 4-qubit code is the most basic error detection circuit. This encodes two logical qubits into four physical qubits and allows for the detection of a single error on any of the two logical qubits. The code's four foundational states are as follows:

$$|00\rangle = \frac{1}{\sqrt{2}}\left(|0000\rangle + |1111\rangle\right)$$

$$|01\rangle = \frac{1}{\sqrt{2}}\left(|1100\rangle + |0011\rangle\right)$$

$$|10\rangle = \frac{1}{\sqrt{2}}\left(|1010\rangle + |0101\rangle\right)$$

$$|11\rangle = \frac{1}{\sqrt{2}}\left(|01110\rangle + |1001\rangle\right)$$

4.2 : Shor nine-qubit code

Shor nine-qubit error correcting code is mainly based on the three-qubit repetition code. This code has the ability to protect a logical qubit from any error such as bit flip error, phase flip error or both error at the same time. Therefore, this code is sufficient to correct for any continuous linear combination of error on a single qubit.

We begin our code by encode for phase flips:

$$|0\rangle \rightarrow |{+}{+}{+}\rangle \quad \text{and} \qquad |1\rangle \rightarrow |{-}{-}{-}\rangle$$

and then encode to check for bit-flip errors

$$|+\rangle \rightarrow {}^{1}\!/\!_{\sqrt{2}}\left(|111\rangle+|000\rangle\right) \text{ and } |-\rangle \rightarrow {}^{1}\!/\!_{\sqrt{2}}\left(|111\rangle-|000\rangle\right)$$

The result of code is as follow :

$$|0\rangle \rightarrow \frac{1}{2\sqrt{2}}\left[\left(|000\rangle+|111\rangle\right)\left(|000\rangle+|111\rangle\right)\left(|000\rangle+|111\rangle\right)\right]$$

$$|1\rangle \rightarrow \frac{1}{2\sqrt{2}}\left[\left(|000\rangle-|111\rangle\right)\left(|000\rangle-|111\rangle\right)\left(|000\rangle-|111\rangle\right)\right]$$

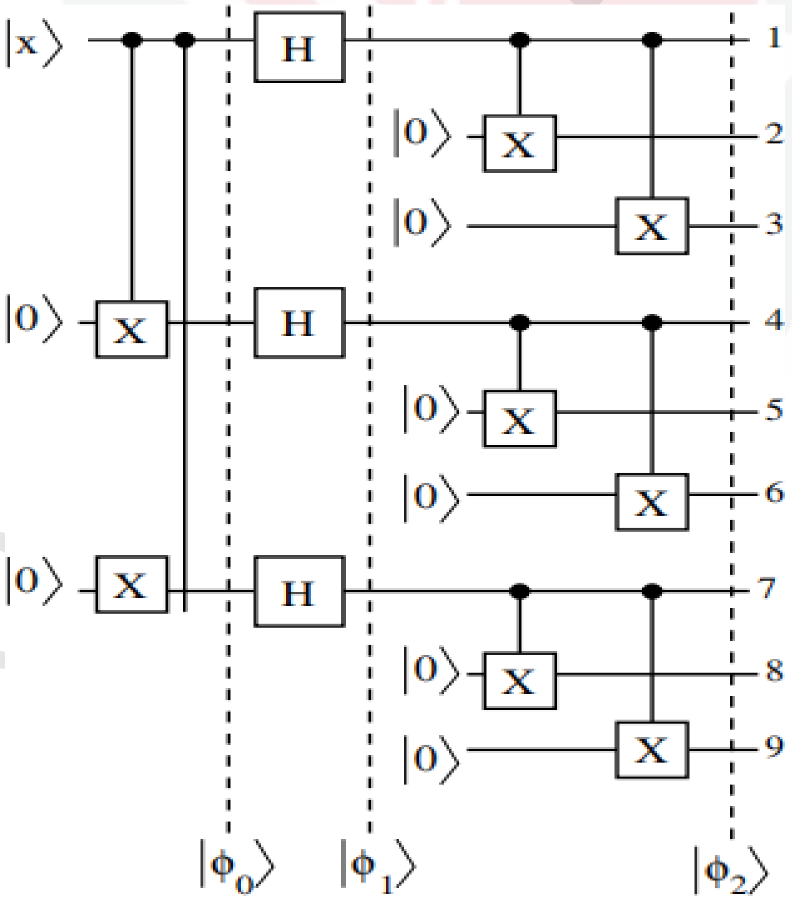


Figure 4.1 : Circuit for 9-qubit error correction

Below are all the stabilizers that are used in this error correction code

$$M_1 = Z_1Z_2, M_2 = Z_2Z_3, M_3 = Z_4Z_5\ , M_4 = Z_5Z_6\ , M_5 = Z_7Z_8\ , M_6 = Z_8Z_9$$

$$M_7 = X_1X_2X_3X_4X_5X_6 \quad and \quad M_8 = X_4X_5X_6X_7X_8X_9$$

It is to be noted that this code is organized into three blocks in which each block comprises of three qubits. so, for the first block contain qubit 1,2 and 3, the second block contain qubit 4,5 and 6 and lastly the third qubit contain qubit 7,8 and 9. This mean at the first block M_1 *and* M_2 will be acted on while M_3 *and* M_4 will act on the second qubit and lastly M_5 *and* M_6 will act on the third qubit.

We'll observe now that the M_i has the properties:

1. They all square to unity which mean the value of the eigenvalues are ±1.
2. They both commute. all of the stabilizers for Z operator and also all the stabilizers for X operator , simply commute with each other.
3. For any stabilizers, the eigenvalue of the uncorrupted codewords $|0\rangle$ and $|1\rangle$ is +1.

We begin with M_1 *until* M_2 , which are stabilizers utilizing z operators. The ancilla qubits x_1 will measure $M_1 = z_1 z_2$ while x_2 will measure $M_2 = z_2 z_3$ and hence detect a bit-flip error in the first group of three qubits in the 9-qubit encoding in the same manner that the 3-qubit, bit-flip code depicted in Fig. 4.1 does. Similarly, x_2 and x_3 detect a bit-flip error in the second and third groups of three qubits (qubits 4–6), respectively.

4.3 : Fault Tolerant Quantum Computer

So far, we have discovered that quantum error correction can be simply accomplished by creating quantum gates that will function flawlessly without any extra faults. This is plainly impossible since quantum computing is constantly prone to additional mistake. Given the number of gates in Shor's 9-qubit syndrome-detection code, we may anticipate that this circuit will create more errors than it will fix. It is also critical that the circuit does not propagate a mistake in one qubit across several qubits, which would be far more difficult to rectify. A "fault tolerant" circuit does not propagate defects.

Below is just the basic main idea of fault tolerant quantum computer

1. The encoded data should be rectified on a regular basis to catch and eliminate mistakes before they accumulate.

2. Error correcting circuits should not propagate mistakes as well.

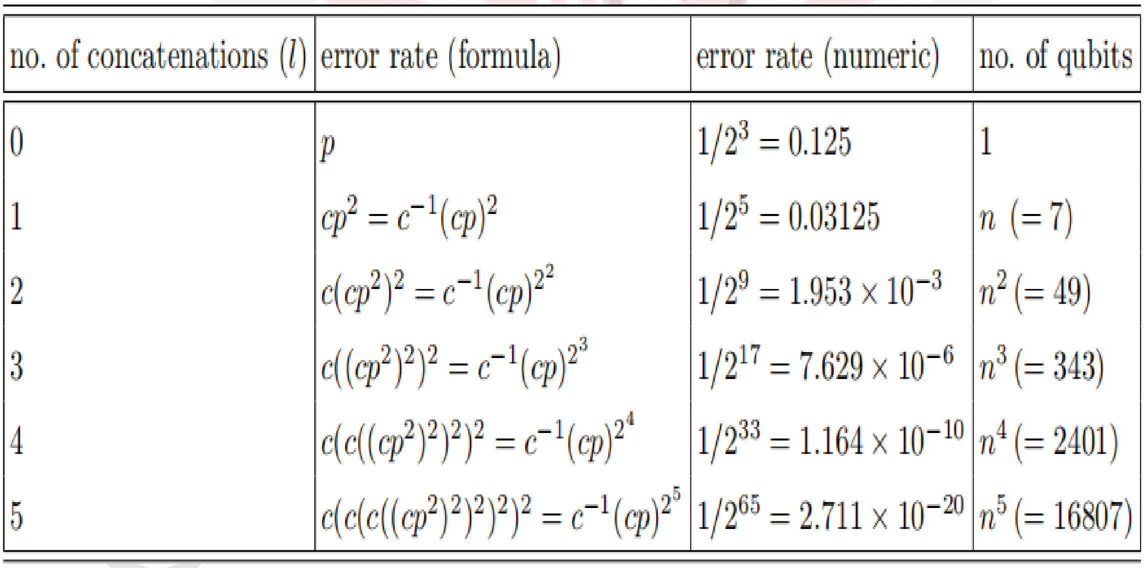
3. Never attempt to decipher quantum information. All operations must be performed on the encoded data.
4. Quantum circuits working on encoded data should be error tolerant. The circuits should not propagate a correctable fault until it becomes a correctable error.

Assume we have an intrinsic error rate of p and a fault tolerant error correction method that corrects 1-qubit mistakes. This indicates that after error correction, the error rate is cp 2 for some constant c. If pc

we have reduced the number of mistakes, hence the threshold error rate is $p_c = \frac{1}{c}$. How can we reduce mistakes even further? Assume that the error correcting process necessitates n physical qubits for every logical qubit. We can then use the same algorithm to error correct each of the n qubits. Concatenation is the name given to this process. Then we have n^2 physical qubits with an error rate of $c\left(cp^2\right)^2 = c^{-1}\left(cp\right)^2$.

In general, if we concatenate l times, the number of qubits is nl, and the error rate is $c^{-1}\left(cp\right)^2$ It is worth noting that, while the number of qubits grows exponentially with the degree of concatenation l, the error rate grows twice exponentially with l.

Table 4.1 Show error rate for different number of concatenations(Young, n.d.)



no. of concatenations (l)	error rate (formula)	error rate (numeric)	no. of qubits
0	p	$1/2^3 = 0.125$	1
1	$cp^2 = c^{-1}(cp)^2$	$1/2^5 = 0.03125$	$n\ (=7)$
2	$c(cp^2)^2 = c^{-1}(cp)^{2^2}$	$1/2^9 = 1.953 \times 10^{-3}$	$n^2\,(=49)$
3	$c((cp^2)^2)^2 = c^{-1}(cp)^{2^3}$	$1/2^{17} = 7.629 \times 10^{-6}$	$n^3\,(=343)$
4	$c(c((cp^2)^2)^2)^2 = c^{-1}(cp)^{2^4}$	$1/2^{33} = 1.164 \times 10^{-10}$	$n^4\,(=2401)$
5	$c(c(c((cp^2)^2)^2)^2)^2 = c^{-1}(cp)^{2^5}$	$1/2^{65} = 2.711 \times 10^{-20}$	$n^5\,(=16807)$

These figures seem implausible. They correspond to a threshold value of $pc = \frac{1}{c} = \frac{1}{2}$, which would be substantially smaller in any actual circuit. They do, however, illustrate, and this is the essential point, that the error rate decreases far quicker than the number of physical qubits increases. Of course, the number of physical qubits per logical qubit must still be quite huge in order to reduce the error rate to an acceptable level for calculation.

4.4 Surface Code:

For numerous reasons, the surface code for quantum error correction is an excellent error correction model. It can be implemented on architectures that only enable the coupling of closest neighbor qubits (rather than the unrestricted long-distance coupling of qubits in various parts of the computer) since it is defined across a 2-dimensional lattice of qubits. The surface code also has one of the greatest fault-tolerant thresholds of any quantum error correction technique, with recent simulations estimating a threshold close to 1%. Finally, the surface code can naturally rectify troublesome error channels such qubit loss and leakage.

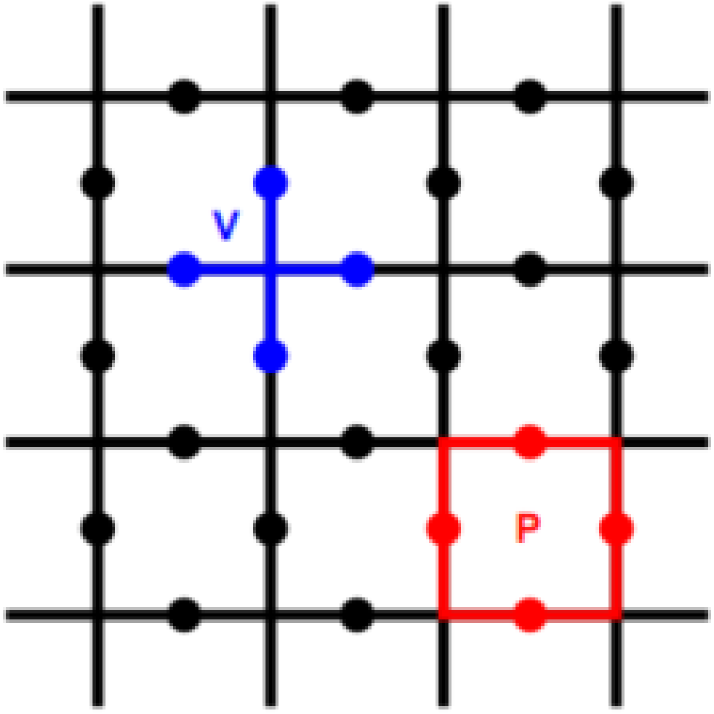


Figure 4.2 : Simple diagram of a surface code

The surface code's qubit lattice. The qubits are situated on the edges, and the stabilisers are made up of two types of local operators. x operators are found near plaquettes (p in the picture), whereas z operators are found near vertices (v in the figure).

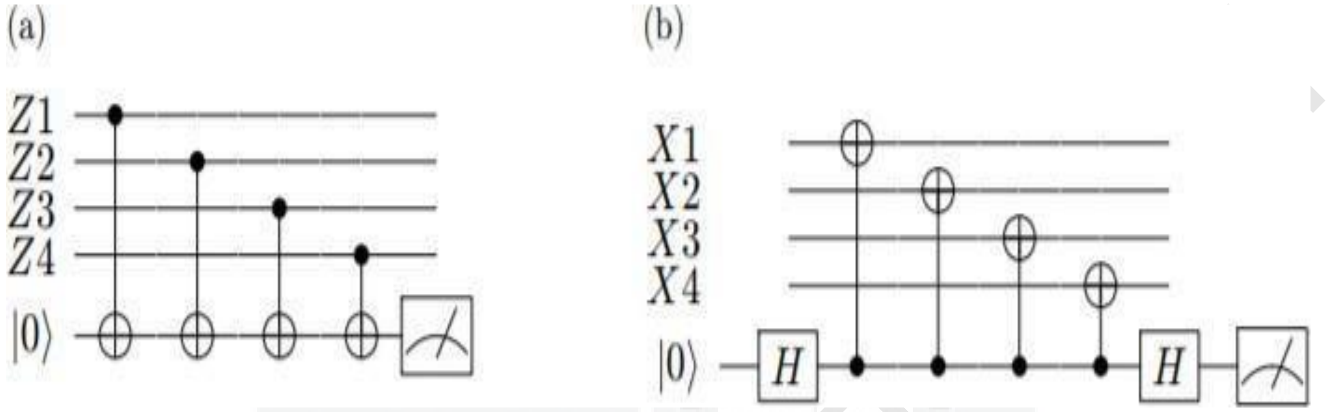


Figure 4.3 : Circuits for syndrome extraction: (a) z syndrome and (b) x -syndrome. Measurements are in the computational basis (Horsman et al., 2012).

Chapter 5

Conclusion

This review should have given you a good overview of some of the most essential theoretical features of quantum error correction and fault-tolerant quantum computation. The main purpose of this discussion was not to give a formal theoretical foundation for QEC and fault-tolerance, but rather to demonstrate the majority of the key principles, conclusions, and strategies that have emerged from this topic. We not only reviewed the fundamentals of QEC with particular examples, but we also briefly examined how we can fix quantum computer error. The reason why scientists are now focusing on developing quantum computer is because this computer can greatly help us in our life

In today digital internet scoop, our entire lives are online and the things that keep our info secure whether it be our credit card we are inputting on Shopee, or our personal photos uploaded to the cloud everything is secure using a common technique across the board encryption and public key cryptography that have been proven time and time again to be a successfully way to digitally secure our data. These technique work would take a modern classical computer century to crack one of these single cryptographic keys. The capacity to model huge complex molecules is a key bottleneck in today's conventional computers. With the ability to simulate large complex molecules, scientists may build all kinds of environments within the simulation to better evaluate drugs and their effects on human bodies and health. However, quantum computer right now is not yet perfect as the classical computer. The difficulty in controlling and manipulating quantum states, which makes encoding quantum information challenging, as well as the sensitivity of such information to disturbance from the environment, pose challenges. Nonetheless, scientists like those at IBM are pushing the boundaries of present knowledge to make quantum computing a reality. The optimal encoding and decoding algorithms for quantum processing are almost certainly yet to be developed. As bigger and less noisy quantum computers become accessible, we will be able to test the performance of error correction concepts on actual machines more often, allowing us to build on the spectacular theoretical gains that we have already witnessed. In my opinion I believe that in the next years, we will see quantum computers with a small number of logical qubits that conduct error correction

5.2: Recommendation for Future Research

One could extend the quantum error correction code for a bigger number’s qubit such as 100 qubits and above.This is because up to now there are not much paper with the perfect example on how to fix a quantum computererror that has more than 10 qubit and above. Beside that one can further research on the effect of the surroundingstoward quantum error correction because went we want to use this quantum computer in our daily life we must first dealt with the surrounding

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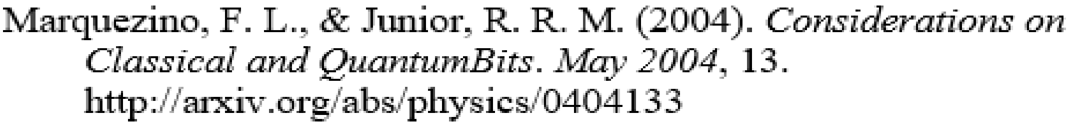
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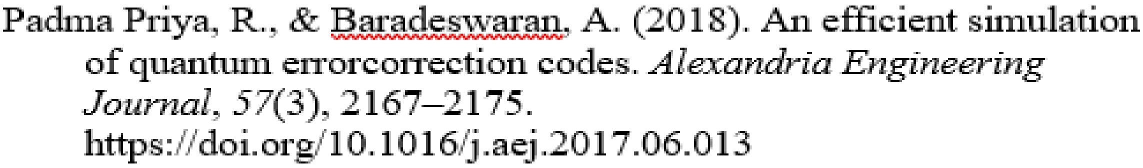
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